The triangles ABK and AMC are similar, hence $AB : AK :: AM : AC$, or $AD : AK :: AM : AF$.

Since the $\angle DAK = \angle MAF$, the triangles DKA and MFA are similar, and $\angle ADK$ is equal to $\angle AMF$.

$\therefore$ DH is parallel to FM...(2).

$\therefore$ DK, FM and EL are parallel. Q. E. D.

IV. **Solution by CHARLES C. CROSS, Libertytown, Md.**

Draw the figure as indicated in the problem.

Let $\angle BLE = x$, $\angle CEL = y$, $\angle DKB = z$, $\angle ADK = w$, $\angle CEM = v$, and $\angle FMC = w$.

$\angle ECL = 180° - (120° + C) = 60° - C$.

Similarly, $\angle EBL = A - 60°$, and $\angle KAD = 60° - A$.

$\angle BCL = 180° - (60° + C) = 120° - C$.

Similarly, $\angle LBC = 120° - B$, and $\angle BAK = 120° - A$.

Hence $\angle BLC = B + C - 60°$, and $\angle BKA = B + A - 60°$.

$\angle BLE + \angle BLC + \angle CEL + \angle ECL = 180°$; by substitution $B + x + y = 180°$...(1).

$\angle BKA + \angle BKD + \angle KDA + \angle KAD = 180°$; by substitution $B + w + z = 180°$..(2).

From (1) and (2), $x + y = w + z$...(3).

If EL and DK are parallel, angle DKB=angle BEL, and angle BLE= angle KDB, or $z = 60° + y$ and $x = 60° + w$. Substituting in (3), $60° + w + y = 60° + w + y$. Hence EL and DK are parallel.

Angle CFM+angle CMF+angle $FCL = 180°$; by substitut'n $v + w - C = 120°$..(4).

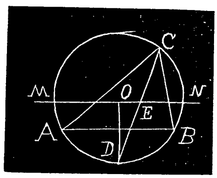
If EL and FM are parallel, then angle MFC=angle ELC, and angle EMC =angle CEL, or $v = x + A + C - 60°$, and $w = y$. Substituting in (4), $A + x + y = 180°$ Since by (1) this relation is true, hence EL and FM are parallel.

107. **Proposed by T. W. PALMER, A. M., Professor of Mathematics, University of Alabama.**

Construct a triangle, given base, vertical angle and radius of inscribed circle.

I. **Solution by H. C. WHITAKER, Ph. D., Professor of Mathematics, Manual Training School, Philadelphia, Pa.**

Denote the base by AB, the vertex by C, and the incenter by O. The angle AOB equals $90° + \frac{1}{2}C$ and hence one locus for O is the arc of a segment capable of containing this angle. Another locus is a parallel to the base the in-radius away. Hence the incircle can be constructed ; AC and BC are then drawn tangent to it.



II. **Solution by J. SCHEFFER, A. M., Hagersfown, Md.**

Describe on the given base AB a circle the upper segment of which contains the given vertical angle. From the center O of this circle let fall the perpendicular on AB and produce it to D. At a distance from AB equal to the given radius of the inscribed circle draw MN parallel to AB. From D as a center with a radius equal to BD draw an arc cutting MN at E, connect E with D and extend DE until it

cuts the circumference at C, then will ABC be the required triangle. For, since $DE=BD$, 2 angle $EBD=180^\circ-$angle $EDB=180^\circ-A$.

$\therefore$ Angle $EBD=90^\circ-\frac{1}{2}A$, but angle $EBA=$angle $EBD-$angle $ABD=$angle $EBD-\frac{1}{2}C=90^\circ-\frac{1}{2}A-\frac{1}{2}C=\frac{1}{2}B$.

$\therefore$ BE is the bisector of B, and by construction, CD is the bisector of C.

$\therefore$ E is the center of the inscribed circle. Q. E. D.

Also solved by *G. B. M. ZERR*, *P. S. BERG*, *COOPER D. SCHMITT*, *F. H. POWE*, *F. W. HAMAWALT*, *ELMER SCHUYLER*, and the *PROPOSER*.

CALCULUS.

81. Proposed by J. OWEN MAHONEY, B. E., M. Sc., Instructor in Mathematics, Carthage High School, Carthage, Texas.

Solve : $y^2(d^2y/dx^2)+a(dy/dx)^2=bx$.

No solution of this problem has been received.

82. Proposed by ALOIS F. KOVARIK, Instructor in Mathematics and Physics, Decorah Institute, Decorah, Iowa.

A pole 60 feet high stands vertically in a river 20 feet deep. How many feet above the surface of the water must it break so that the top bending down would touch the bottom and the distance on the surface of water between the points where the parts of the pole enter the water would be a maximum?

I. Solution by C. HORNUNG, A. M., Professor of Mathematics, Heidelberg University, Tiffin, O., and GUY B. COLLIER, Union College, B. S. Course, Schenectady, N. Y.

Let $x=$the number of feet above the surface of the water the pole must break, and $y=$the number of feet between the parts of the pole on the surface of the water, which is to be a maximum.

By similar triangles we find $y=\dfrac{2x}{x+20}\sqrt{300+30x}$.

Simplifying and placing the first derivative equal to zero, we have a biquadratic in x whose roots are : 0, -20, 6.055, and -66.055. By substitution in the second derivative we find that 6.055 is the only one of these roots that renders y a maximum. Therefore $x=6.055$ is the required result.

II. Solution by J. SCHEFFER, A. M., Hagerstown, Md.

Let ABC represent the pole, BC being the part under water. Let D be the point where it breaks off, so that $DA=DE$. Let $AB=a$, $BC=b$, $BD=x$, $BF=y$; then $DA=DE=a-x$. $CE=\sqrt{[(a-x)^2-(b+x)^2]}=\sqrt{(a+b)}.\sqrt{(a-b-2x)}$ and $CE:y=b+x:x$, whence $y=\sqrt{a+b}.\dfrac{x}{b+x}\sqrt{a-b-2x}$.

$\therefore$ $M=\dfrac{x^2}{(b+x)^2}(a-b-2x)$ is to be a maximum.

By differentiation we obtain after all the necessary and simple transformations the quadratic $x^2+3bx=(a-b)b$, whence $x=\frac{1}{2}[-3b+\sqrt{(5b^2+4ab)}]$.

For the numerical value $a=40$, $b=20$, we get $x=10(\sqrt{13}-3)=6.055$.